**Homework 5**

**Instructions:** Do as many of the problems as you like, but make sure to complete at least **three**. Then I will create a solution from your work.

1. Find an example of numbers *a, b* and *n* such that but
2. Suppose that where Show that if and only if and

* If then and
  + by the definition of congruence.
  + Since then and .
  + Thus, by the definition of congruence, and .
* If and then .
  + and by the definition of congruence. So and .
  + Thus, is a common multiple of and and so .
  + Since and we know that then .
  + So and by the definition of congruence.

1. Find a complete residue system modulo 5 composed entirely of multiples of 9.

* A complete residue system modulo 5 composed entirely of multiple of 9 is the one formed by the first 5 nonnegative multiples of 9: {0, 9, 18, 27, 36}
* I know this is a complete residue system because if you find the least residue of each of those numbers you get {0, 4, 3, 2, 1}, which is the least residue system modulo 5 which shows us that each residue class is accounted for.

1. Show that a perfect square must have one of 0, 1, 4, 5, 6, or 9 for its units digit.

* Looking at just the units digit is equivalent to taking the least residue of each number modulo 10.
* By the division algorithm, any integer can be written in the form for . Thus, the square of any integer will be of the form for .
* This means that for integer , if for then and thus we need only check the squares of the least residue system modulo 10 to prove the statement.
* Removing multiple entries yields the set of possible residues modulo 10 (the units digits) of a perfect square to be .

1. Show that any *n* consecutive integers form a complete residue system modulo *n*.
2. Show that is not a complete residue system modulo *m* if

* Since there must be unique residue classes in any complete residue system modulo , to show that (which has entries) is not a complete residue system modulo , it is enough to show that at least two of the entries in that list are part of the same residue class modulo .
* Notice that and thus . Additionally, .
* Since we know so we have found 2 different entries of the proposed residue system which each refer to the residue class of 1 modulo .
* Thus, there are not enough unique residue classes represented by the proposed residue system to constitute a complete residue system.
* This argument works for multiple other entries as well if is larger. For example because and Through repeating this argument we can conclude that the proposed residue system only includes unique residue classes.

1. Prove that 17 does not divide for any *n.*

* Every can be represented as for .
* Thus, every perfect square can be represented as for .
* So the possible residues of modulo 17 are exactly the squares of the possible residues of modulo 17.
* Squaring each entry of and reducing mod 17 yields
* We only needed to check half of this list because the entries will repeat after the ninth entry for the reason presented at the end of problem 6. But I calculated all squares anyway.
* Removing repeated entries tells us that the possible residues of modulo 17 are .
* Multiplying each of these entries by 5, adding 15, then reducing modulo 17 yields .
* The fact that this list does not contain a 0 means that is never congruent to 0 modulo 17. Thus 17 does not divide for any